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Technical Note

Bulk mean temperature in porous medium analysis

Seok Pil Jang [a,b], Sung Jin Kim [c], Stephen U.S. Choi [b,*]

[a] School of Aerospace and Mechanical Engineering, Hankuk Aviation University Goyang, Geyeonggi-do, 412-791, South Korea
[b] Energy Technology Division, Argonne National Laboratory Argonne, IL 60439, USA
[c] Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305-701, South Korea

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1. Introduction

Heat transfer in porous media has been the subject of many investigations due to the increasing interest in chemical catalytic reactors, building thermal insulation, heat exchangers, petroleum reservoirs, geothermal operations, and so on. In addition to these conventional applications, new applications have been found in the emerging field of microscale heat transfer [1]. For example, smaller flow passages and fins are needed for compact heat exchangers and electronics cooling, as Bejan [2] has noted. As the dimensions get smaller, classical flow structures approach a limiting case that is much better suited for porous medium analysis [2]. Koh and Colony [3], Tien and Kuo [4] and Kim and co-workers [1,5] have modeled microscale structures as a porous medium. The pores in porous media have a wide range of sizes from several hundred micrometers in sintered metals to lager than one millimeter in packed beds.

For both conventional and new applications, porous medium analysis has been used to understand the thermal characteristics of porous media as well as microscale structures by calculating the Nusselt number, the overall heat transfer coefficient, or the thermal resistance. To make these calculations, the bulk mean temperature, $\langle T_b \rangle$, should be determined:

$$\langle T_b \rangle = \left\langle \frac{1}{u_m A_C} \int T u \, dA_C \right\rangle \tag{1}$$

where A_C, T, u, u_m and $\langle\rangle$ are cross-sectional area, temperature, velocity, mean velocity and volume-averaged value. Because it is very difficult to calculate the bulk mean temperature in porous media using Eq. (1), many investigators [5–9] have calculated the bulk mean temperature using Eq. (2) in their analyses.

$$\langle T_b \rangle_{\text{previous}} = \frac{1}{\langle u_m \rangle A_C} \int \langle T \rangle \langle u \rangle \, dA_C \tag{2}$$

However, results from Eq. (2) are not identical to those from Eq. (1). Therefore, the Nusselt number, overall heat transfer coefficient or thermal resistance that is calculated from Eq. (2) may not be valid for many practical applications [10].

In this paper, a model for determining the bulk mean temperature in porous medium analysis is theoretically derived based on the spatial decompositions of velocity and temperature presented by Carbonell and Whitaker [11]. Then, we compare calculations from our model with numerical results obtained from classical momentum and energy equations. This comparison shows that our proposed model accurately predicts the bulk mean temperature in porous medium analysis. In addition, we show that the previous model, which has been generally used by many researchers, is invalid except for select cases and identify important parameters that bring about the difference between the proposed model and the previous model.

2. Model for the bulk mean temperature in porous media

The bulk mean temperature in porous medium analysis is defined as follows:

* Corresponding author. Tel.: +1 630 252 6439; fax: +1 630 252 5568.
E-mail address: Choi@anl.gov (S.U.S. Choi).

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$$\langle T_b \rangle = \left\langle \frac{1}{u_m A_C} \int Tu \, dA_C \right\rangle = \frac{1}{\langle u_m \rangle A_C} \left\langle \int Tu \, dA_C \right\rangle = \frac{1}{\langle u_m \rangle A_C} \int \langle Tu \rangle \, dA_C \quad (3)$$

Because the velocity of the solid phase is zero, Eq. (3) can be rewritten as

$$\langle T_b \rangle = \frac{1}{\langle u_m \rangle A_C} \int \langle T_\beta u_\beta \rangle \, dA_C \quad (4)$$

where the subscript β denotes the fluid phase in a porous medium. To determine $\langle T_\beta u_\beta \rangle$, we use the spatial decompositions of velocity and temperature presented by Carbonell and Whitaker [11]:

$$u_\beta = \langle u_\beta \rangle^\beta + \tilde{u}_\beta, \quad T_\beta = \langle T_\beta \rangle^\beta + \tilde{T}_\beta \quad (5)$$

where $\tilde{u}_\beta$ and $\tilde{T}_\beta$ are the spatial deviation of velocity and temperature from the averaged value; $\langle \rangle^\beta$ is the volume-averaged value of β phase. By using Eq. (5), we can estimate the value of Eq. (4):

$$\langle T_b \rangle = \frac{1}{\langle u_m \rangle A_C} \left[\int \left\langle \langle T_\beta \rangle^\beta \langle u_\beta \rangle^\beta \right\rangle dA_C + \left\langle \langle T_\beta \rangle^\beta \tilde{u}_\beta \right\rangle dA_C + \left\langle \tilde{T}_\beta \langle u_\beta \rangle^\beta \right\rangle dA_C + \langle \tilde{T}_\beta \tilde{u}_\beta \rangle \, dA_C \right] \quad (6)$$

Since $\langle \tilde{T}_\beta \rangle = 0$ and $\langle \tilde{u}_\beta \rangle = 0$, Eq. (6) can be rearranged as

$$\langle T_b \rangle = \frac{1}{\langle u_m \rangle A_C} \left[\int \left\langle \langle T_\beta \rangle^\beta \langle u_\beta \rangle^\beta \right\rangle dA_C + \langle \tilde{T}_\beta \tilde{u}_\beta \rangle \, dA_C \right] \quad (7)$$

Furthermore

$$\int \left\langle \langle T_\beta \rangle^\beta \langle u_\beta \rangle^\beta \right\rangle dA_C = \int \langle T_\beta \rangle \langle u_\beta \rangle \, dA_C \quad (8)$$

Based on Eqs. (7) and (8), we can obtain a relation between Eqs. (1) and (2) as follows:

$$\langle T_b \rangle = \langle T_b \rangle_{\text{previous}} + \frac{1}{\langle u_m \rangle A_C} \int \langle \tilde{T}_\beta \tilde{u}_\beta \rangle \, dA_C \quad (9)$$

The second term on the right-hand side of Eq. (9) can be theoretically derived with the simple assumption that the pore within a porous medium has a small circular geometry, and that the fluid flow is hydrodynamically and thermally fully developed. These assumptions seem appropriate because new model calculations agree well with numerical results, as shown in Figs. 2 and 3. From the analytical solutions of velocity and temperature for a circular tube with constant heat flux boundary condition, the spatial deviations of velocity and temperature are expressed by

$$\tilde{u}_\beta = 2\langle u_\beta \rangle^\beta \left(1 - \frac{r^2}{R^2}\right) - \langle u_\beta \rangle^\beta,$$
$$\tilde{T}_\beta = -\frac{q''_p}{k_f} d_p \left[0.208 - \frac{1}{2}\frac{r^2}{R^2} + \frac{1}{8}\frac{r^4}{R^4}\right] \quad (10)$$

where k_f, d_p, q''_p, r, and R are the heat transfer coefficient, Nusselt number, heat flux dissipated in a pore, radial distance, and pore radius, respectively.

By substituting Eq. (10) into Eq. (9), we obtain

$$\langle T_b \rangle = \langle T_b \rangle_{\text{previous}} - 0.0625 \frac{q''_p}{k_f} d_p \quad (11)$$

where d_p and k_f are pore diameter and fluid thermal conductivity, respectively. Parameters, which differentiate Eq. (1) from Eq. (2) are heat flux dissipated in a pore, pore diameter and fluid thermal conductivity. As the heat flux dissipated in a pore is increased, the pore diameter increases or the fluid thermal conductivity decreases, the difference between Eqs. (1) and (2) becomes larger. Therefore, Eq. (2) should not be used to predict the bulk mean temperature except for specific cases where the heat flux dissipated in a pore is very small, pore diameter is very small, or the fluid conductivity is very large.

3. Validation

To check the validity of our proposed model for the bulk mean temperature of a porous medium, Eq. (11), we numerically investigated two ideal porous media: the fin structures shown in Fig. 1 and a microchannel structure [5,12]. We compare the results calculated from Eq. (11) with those from Eq. (2), as well as classical simulation results.

3.1. Fin structures

The problem under consideration is the parallel flow through the fin structured channel shown in Fig. 1(a). The air flows along the x-axis. The bottom surface is kept at constant heat flux, and the top surface is insulated. An REV (representative elementary volume) for volume-averaging is shown in Fig. 1(b). For the numerical domain, the no-slip boundary condition is prescribed at the top and bottom surfaces, and the symmetric condition is specified on two sides located normal to the y-axis. Porosity is 2/3, and size of a fin is $2\,\text{cm} \times 2\,\text{cm} \times 10\,\text{cm}$. By using the finite-volume method, we can numerically calculate the bulk mean temperature and compare the bulk mean temperature with that from Eq. (11). Fig. 2 shows that results from Eq. (11) match closely with classical numerical results. On the other hand, results based on the previous model, Eq. (2), produce significant errors in predicting bulk mean temperature.

3.2. Microchannel structure

To further validate the proposed model, we compare the thermal resistance calculated from Eq. (11) with re-

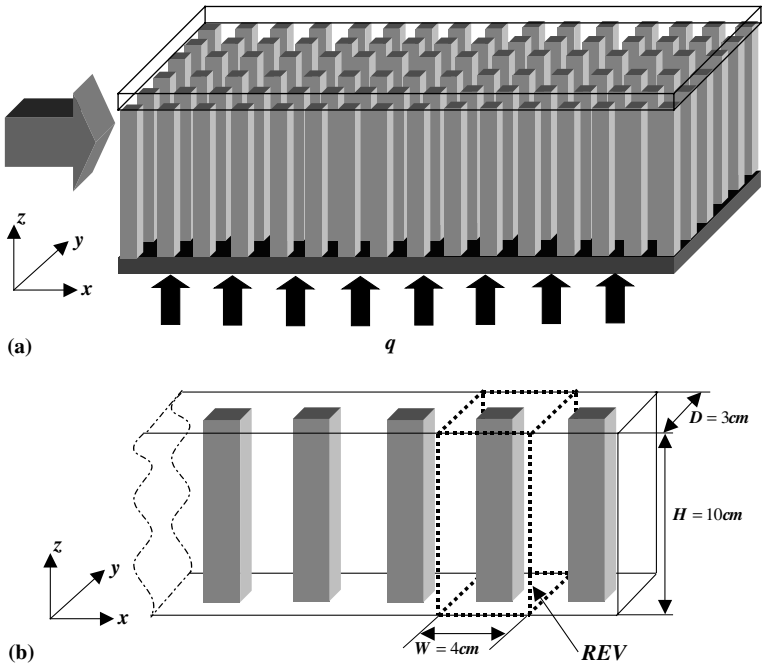

Fig. 1. Fin structures as an ideal porous medium. (a) Physical domain. (b) Numerical domain.

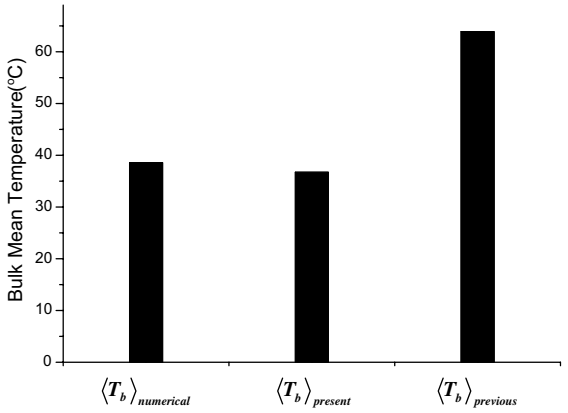

Fig. 2. Bulk mean temperatures in a fin structure obtained from classical numerical results and from present and previous models.

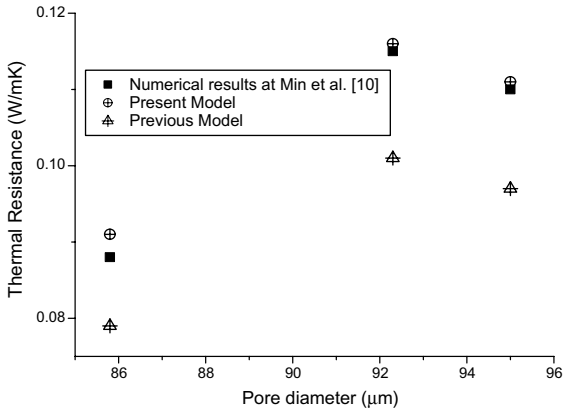

Fig. 3. Comparison of thermal resistances for the microchannel structures.

sults from Eq. (2), as well as classical numerical results of Min et al. [12]. The thermal resistance for the microchannel structure is defined as

$$\theta = \frac{\langle T_w \rangle - \langle T_{\text{in}} \rangle}{q} = \frac{\langle T_w \rangle - \langle T_{\text{b}} \rangle}{q} + \frac{1}{\dot{m} C_{\text{p}}} \tag{12}$$

where T_w, T_{in}, q, $\dot{m}$, and C_{p} are maximum temperature at the base of the microchannel structure, inlet temperature, heat transfer rate, mass flow rate, and heat capacity, respectively. Fig. 3 shows that the proposed model predicts accurately the numerical results of Min et al. [12], while the results from Eq. (2) have a large error of 12.2%.

Having confirmed the validity of the proposed model, we conducted a parametric study to quantitatively evaluate the difference between the proposed model, Eq. (11), and the previous model, Eq. (2). Fig. 4 shows that the two models differ substantially for a practical range of microstructure size, fluid thermal property, and heat flux. For example, Fig. 4(a) shows

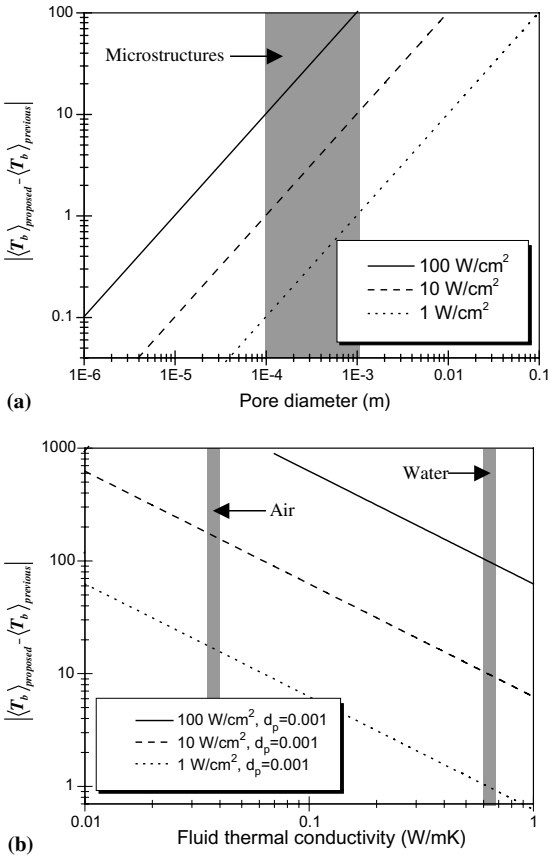

Fig. 4. Parametric results for $|\langle T_b\rangle_{\text{proposed}} - \langle T\rangle_{\text{previous}}\rangle|$ as function of pore diameter, heat flux, and fluid thermal conductivity. (a) Effect of a pore diameter. (b) Effect of fluid thermal conductivity.

that the two models may yield up to about 10 °C difference in the bulk mean temperature for water at a heat flux of 10 W/cm^2 and pore diameter of 1 mm. Fig. 4(b) shows that the two models give up to about 20 °C difference in the bulk mean temperature of air at a heat flux of 1 W/cm^2. Considering that the heat flux on microscale structure cooling devices is continually increasing due to trends toward faster speeds and smaller features for microelectronic devices, more power output for engines, and brighter beams for optical devices, the results from the parametric study clearly show the advantage of the new model over the previous model in predicting the bulk mean temperature.

4. Conclusion

In this paper, we propose a new model for calculating the bulk mean temperature in a porous medium. The model, Eq. (11), is theoretically derived by using the spatial decomposition of velocity and temperature presented by Carbonell and Whitaker [10]. Calculated results with new model match closely with classical numerical results. Furthermore, the previous model, Eq. (2) cannot be used to predict the bulk mean temperature in a porous medium except for select cases. Finally, we identify important parameters which bring about the difference between the previous model, Eq. (2) and a new model, Eq. (11) and quantitatively evaluate the difference. Because the two models differ substantially for a practical range of microstructure size, fluid thermal property, or heat flux, the new model has the clear advantage over the previous model in accurately predicting the bulk mean temperature for all these parameters.

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